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Analysis of Soil-Structure Interaction in the Frequency Domain

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ABSTRACT

In practical soil-structure analysis a number of basic issues need to be addressed. First, the soil-structure interaction problems are generally of a very large size. A suitable transformation should therefore be used to reduce the size of the problem. Second, analysis in the frequency domain, which is the preferred method for problems involving wave propagation in an infinite soil medium, usually requires the application of discrete Fourier transformation. The use of such transformation often leads to unacceptable errors caused by aliasing or overlapping. Analytical techniques that address these issues are presented here. A set of Ritz vectors based on the concept of component mode synthesis is developed for the transformation of the interaction problem. The method of artificial damping, developed by two of the authors, and reported earlier, is applied in the solution. The effectiveness of these techniques is illustrated by means of a simple example of soil-structure interaction.

INTRODUCTION

Seismic analysis of soil-structure interaction problems based on substructure method is usually carried out in the frequency domain. This is because frequency domain analysis is most effective in treating the problem of wave propagation in the semi-infinite regular soil region. Also, with the use of Fast Fourier Transform (FFT), frequency domain analysis becomes computationally very efficient. Finally, when the Boundary Element Method (BEM) is used in the analysis, the fundamental solution for the soil region is much simpler in the frequency domain than in the time domain.

Although many researchers have studied the frequency domain analysis of soil-structure interaction, a number of problems still exist in the analysis procedure. The soil-structure interaction problems are normally of very large size. This is partly because the soil region is of an infinite extent; as a result, its discretization contributes a large number of degrees of freedom. A proper modelling of the soil region to minimize the degrees of freedom in association with a suitable transformation is required to reduce the size of the problem. Mode shapes of the soil-structure system have been used in the past as Ritz vectors defining the transformation. However, when BEM is used to model the soil, the resulting transformed matrices are unsymmetric and the transformation method is not particularly efficient. The use of partial mode shapes of the structure on a fixed base along with rigid body displacements of the foundation has been found to give substantial errors (Vaish and Chopra, 1973). A new set of Ritz vectors are used in this

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study and are found to be quite effective. These Ritz vectors comprise fixed-base normal modes of the superstructure and static constraint modes obtained by applying a unit displacement, in turn, to each degree of freedom on the soil structure interface.

Another source of problem in the frequency domain analysis of the interaction problem is the error associated with the use of Discrete Fourier Transforms (DFT), particularly when damping in the system is low. This error is caused by overlapping or aliasing. An unnecessarily long period of time must be used in the analysis to minimize this error. A novel method of avoiding this problem, referred to as the Artificial Damping Method, was published by two of the present authors (Humar and Xia, 1993) and simultaneously by Kausel and Roesset (1992) who called it the Exponential Window Method. The method of artificial damping is applied in the present work for the frequency domain analysis of soil-structure interaction problem and is shown to be very effective and efficient.

FORMULATION OF THE SOIL-STRUCTURE INTERACTION PROBLEM

The formulation of the equation of motion for a soil-structure system has been adequately described in the literature (Vaish and Chopra, 1973; Wolf, 1985). The equations of motion relating the response to a specific free field input can be expressed as (Wolf, 1985)

$$\left(-\Omega^2\begin{bmatrix}\mathbf{M}_{ss} & \mathbf{M}_{sb}\\ \mathbf{M}_{bs} & \mathbf{M}_{bb}\end{bmatrix}+i\Omega\begin{bmatrix}\mathbf{C}_{ss} & \mathbf{C}_{sb}\\ \mathbf{C}_{bs} & \mathbf{C}_{bb}\end{bmatrix}+\begin{bmatrix}\mathbf{K}_{ss} & \mathbf{K}_{s}\\ \mathbf{K}_{bs} & \mathbf{K}_{bb}+\mathbf{S}_{ff}(\Omega)\end{bmatrix}\right)\begin{bmatrix}\mathbf{V}_s(\Omega)\\ \mathbf{V}_b(\Omega)\end{bmatrix}=-\begin{bmatrix}\mathbf{M}_{ss} & \mathbf{M}_{sb}\\ \mathbf{M}_{bs} & \mathbf{M}_{bb}\end{bmatrix}\begin{bmatrix}\ddot{\mathbf{V}}_{s0}\\ \ddot{\mathbf{V}}_{b0}\end{bmatrix} \tag{1}$$

where, **M**, **C** and **K** are respectively the mass, damping and stiffness matrices for the superstructure and $\mathbf{S}_{ff}(\Omega)$ is the frequency dependent soil impedance matrix. Subscript s represents the superstructure degrees of freedom, while b represents the base degrees of freedom that lie along the interface between the soil and the structure. Also, $\mathbf{V}_0$ represents the pseudo-static displacements caused by the free-field ground motions while **V** represents the displacements relative to the pseudo-static displacements. The soil impedance matrix, $\mathbf{S}_{ff}(\Omega)$, may be obtained by a finite element or a boundary element method (Xia, 1994). This matrix is in general unsymmetric when the BEM is used.

The dynamic problem represented by Eq. 1 is of large size, its solution therefore involves a large volume of computation. It is possible to reduce the size of the problem by a Ritz transformation given by

$$\mathbf{V}(\Omega)=\mathbf{X}\mathbf{Z}(\Omega) \tag{2}$$

in which **X** is the matrix of Ritz vectors and $\mathbf{Z}(\Omega)$ are the generalized coordinates. The size of **X** is $N \times M$ where, N is the total number of structure degrees of freedom and M is the number of Ritz vectors. The transformed equations are

$$\left[-\Omega^2\mathbf{M}^* + i\Omega\mathbf{C}^* + \mathbf{K}^* + \mathbf{L}(\Omega)\right]\mathbf{Z}(\Omega) = -\mathbf{X}^T\mathbf{M}\ddot{\mathbf{V}}_0 \tag{3}$$

where $\mathbf{M}^* = \mathbf{X}^T\mathbf{M}\mathbf{X}$, $\mathbf{C}^* = \mathbf{X}^T\mathbf{C}\mathbf{X}$, $\mathbf{K}^* = \mathbf{X}^T\mathbf{K}\mathbf{X}$, and

$$\mathbf{L}(\Omega) = \mathbf{X}^T\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{S}_{ff}\end{bmatrix}\mathbf{X} = \mathbf{X}_b^T\mathbf{S}_{ff}\mathbf{X}_b \tag{4}$$

in which $\mathbf{X}_b$ is obtained from **X** by taking the rows corresponding to interface degrees of freedom. The complex frequency response functions denoted by $\mathbf{H}(\Omega)$ is then obtained from

$$\left[-\Omega^2\mathbf{M}^* + i\Omega\mathbf{C}^* + \mathbf{K}^* + \mathbf{L}(\Omega)\right]\mathbf{H}(\Omega) = -\mathbf{X}^T\mathbf{F} \tag{5}$$

where $\mathbf{F}$ is either $\mathbf{F}_h = \mathbf{M}\mathbf{r}_h$ or $\mathbf{F}_v = \mathbf{M}\mathbf{r}_v$, $\mathbf{r}_h$ represents the static displacements caused by a unit horizontal displacement along the free field and $\mathbf{r}_v$ represents the static displacements caused by a unit vertical displacement of the free field.

The Ritz vectors to be used in the transformation of Eq. 1 can be obtained by using the following modified stiffness matrix

$$\hat{\mathbf{K}} = \begin{bmatrix} \mathbf{K}_{ss} & \mathbf{K}_{sb} \\ \mathbf{K}_{bs} & \mathbf{K}_{bb} + \mathbf{S}_{ff}(\epsilon(\Omega)) \end{bmatrix} \tag{6}$$

where $\epsilon(\Omega)$ is a very small value of Ω. The Ritz vectors are the mode shapes generated by solving the eigenvalue problem given by

$$(-\omega^2\mathbf{M} + \hat{\mathbf{K}})\mathbf{X} = \mathbf{0} \tag{7}$$

SUBSTRUCTURE COUPLING USING RITZ VECTORS

When $\mathbf{S}_{ff}$ is obtained by BEM, the modified stiffness matrix $\hat{\mathbf{K}}$ in Eq. 7 is an unsymmetric matrix. Therefore, a half bandwidth storage of $\hat{\mathbf{K}}$ is not possible, neither is the $\mathbf{LDL}^T$ decomposition. It is well known that symmetry of stiffness matrix can lead to considerable saving in computing time required in generating Ritz vectors. It is a drawback of using $\hat{\mathbf{K}}$ that such economy can not be realized. The shortcomings cited above can be avoided if the Ritz reduction is applied only to a part of the soil-structure system. The idea of using mode shapes of a part of the soil-structure system, in particular the mode shapes of the superstructure on a fixed base in association with the displaced shapes produced by rigid body motions of the foundation, has been suggested by several researchers (Vaish and Chopra, 1973; Guiterrez and Chopra, 1978). However, when the foundation is flexible, significant errors are introduced by the use of such mode shapes. The authors believe that these errors are not, in fact, caused by the use of partial mode shapes of the system, but because of the assumption of a rigid body displacement for the degrees of freedom of the foundation. A new technique is used here to avoid such an assumption and an alternative set of effective partial (component) Ritz vectors is presented.

In Eq. 1, the complete stiffness matrix $\mathbf{K}$ is singular with rigid body motion included. However, submatrix $\mathbf{K}_{ss}$ is not singular. Let $\mathbf{X}_s$ be the mode shapes generated by using the stiffness matrix of the superstructure with all the supports fixed. These mode shapes are obtained by solving the following eigenvalue problem

$$(-\Omega^2\mathbf{M}_{ss} + \mathbf{K}_{ss})\mathbf{X}_s = 0 \tag{8}$$

Matrix $\mathbf{K}_{ss}$ is usually a positive definite, symmetric and sparse matrix. These properties allow the use of half bandwidth storage and $\mathbf{LDL}^T$ decomposition and thus greatly enhance the computing efficiency particularly when the order of matrix $\mathbf{K}_{ss}$ is large.

A set of Ritz vectors that contains the partial Ritz vectors $\mathbf{X}_s$ may be expressed as

$$\mathbf{Y} = \begin{bmatrix} \mathbf{X}_s & \mathbf{X}_c \\ \mathbf{0} & \mathbf{I}_b \end{bmatrix} \tag{9}$$

in which matrix $[\mathbf{X}_c \quad \mathbf{I}_b]^T$ represents the displacements at the structure and interface degrees of freedom caused by a unit displacement, applied in turn, to each of the degrees of freedom at the soil-structure interface. The matrix just referred to, in fact, consists of a set of so called constraint modes or Ritz vectors (Craig and Chang, 1976) and are obtained from

$$\begin{bmatrix} \mathbf{X}_c \\ \mathbf{I}_b \end{bmatrix} = \begin{bmatrix} -\mathbf{K}_{ss}^{-1}\mathbf{K}_{sb} \\ \mathbf{I}_b \end{bmatrix} \tag{10}$$

The transformation described above is based on the concept of component mode synthesis and is different from the one used by Vaish and Chopra (1973). The Ritz vectors in Eq. 9 reduce the size of only one substructure keeping the number of degrees of freedom of another substructure (soil foundation) intact. If the number of interface nodes is comparatively large, a two stage reduction can be used. The Ritz vectors to be used in the second stage transformation are evaluated by solving the following eigenvalue problem

$$\left(-\lambda^2\tilde{\mathbf{M}} + \tilde{\mathbf{K}}\right)\mathbf{Q} = \mathbf{0} \tag{11}$$

where $\tilde{\mathbf{M}} = \mathbf{Y}^T\mathbf{M}^*\mathbf{Y}$, $\tilde{\mathbf{K}} = \mathbf{Y}^T\left[\mathbf{K}^* + \mathbf{L}(\epsilon(\Omega))\right]\mathbf{Y}$.

The generation of component Ritz vectors, $\mathbf{X}_s$ in Eq. 9 requires information related to the superstructure only and is not dependent on a knowledge of the properties of foundation soil. Matrix $\mathbf{X}_s$ thus needs to be obtained only once from $\mathbf{M}_{ss}$ and $\mathbf{K}_{ss}$, respectively the mass and stiffness matrices for the superstructure with the interface nodes fixed.

METHOD OF ARTIFICIAL DAMPING

It is well known that frequency domain analysis based on the use of Discrete Fourier Transforms (DFT) does not always provide accurate results for the dynamic response of the structure (Humar and Xia, 1993). If sufficient damping does not exist in the system, the response obtained by using the DFT may be in severe error because of overlapping or aliasing. Soil-structure systems may be subjected to this problem. To ensure that the correct response is obtained for the soil-structure system obtained, the method of artificial damping could be used instead of the standard DFT method. As explained in Humar and Xia (1993), the use of artificial damping requires the evaluation of modified frequency response functions $\hat{H}(\Omega)$ which are related to $H(\Omega)$ by the expression

$$\hat{H}(\Omega) = H(\Omega - ia) \tag{12}$$

where a is an arbitrarily selected parameter and i is the imaginary number. This also requires the evaluation of soil impedance matrix $\mathbf{S}_{ff}$ for the imaginary frequency $\Omega - ia$. When the BEM method is used to obtain soil impedance, all that is required is to evaluate the Bessel functions involved in the analysis for $(\Omega - ia)$ in place of Ω.

An infinite domain of soil does provide a certain amount of damping in the system through radiation of waves to infinity. This implies that the standard DFT method can also be used, provided a sufficiently long period is selected in the analysis. The procedure is, however, inefficient as compared to the method of artificial damping.

EXAMPLE PROBLEM

As an example of the application of techniques described in this paper, the five story frame shown in Fig. 1a is analyzed for its responses to earthquake motion. To focus attention on the effectiveness of the techniques proposed, the soil is represented by concentrated springs and viscous dampers located along the interface degrees of freedom. The soil impedance matrix is, in this case, independent of the excitation frequency.

The frame shown in Fig. 1a has 72 degrees of freedom, three at each node. The following properties are used for all members of the frame: modulus of elasticity $E = 2. \times 10^{10}$ N/m^2, area of cross-section $A = 0.36$ m^2, moment of inertia $I = 0.0108$ m^4, and mass density $\rho = 2400$ kg/m^3. Each horizontal spring representing the soil has a spring constant of 7×10^6 N/m and each vertical spring has a constant of 8×10^6 N/m. The damping constants for the dampers representing the soil are 0.002 times the stiffnesses of the parallel springs. The damping matrix for the superstructure is assumed to be 0.002 times its stiffness matrix. The structure is subjected to a constant free field acceleration of magnitude 10 m/s^2 lasting for 4 s as shown in Fig 1b.

A time domain solution of the structure using the full 72 degrees of freedom. is compared in Fig. 2 with frequency domain solutions obtained by (1) standard DFT technique, and (2) by using the method of artificial damping. The response quantity plotted is the horizontal displacement at Node 1 relative to the free field didplacement. For analysis in the frequency domain, the total time period T_0 is taken to be 12.775 s with a sampling interval of 0.025 s, giving 512 sample points. In the method of artificial damping, a is taken as 0.36. The unit impulse function $h(t)$ obtained by taking the inverse transform of $H(\Omega)$ is truncated at 6.375 s, hence the frequency domain solution is expected to be valid up to 6.375 s. From Fig. 2 it is clear that for the selected value of T_0 the standard DFT method fails due to aliasing errors while the method of artificial damping provides results that are almost indistinguishable from the exact results.

In Fig. 3 the exact time domain solution is compared with the frequency domain solution of a reduced system. Again, the response quantity shown is the relative horizontal displacement of Node 1. The frequency domain solution uses the method of artificial damping. Two different procedures are used in reducing the system. In one procedure 7 normal modes of the structure on a fixed base along with 3 displaced shapes produced by rigid body motions of the foundation are used to transform the equations of motion. The results obtained by this procedure are in serious error. Analysis by this procedure took a cpu time of 159 s on a Sparc ELC workstation. In another procedure 2 fixed base normal modes of the structure are used in association with 12 constraint modes. A second stage transformation using the modes obtained from Eq. 11 is applied and only 7 of the second stage vectors are used in the final analysis. The results obtained by this procedure are almost identical to the exact results. The cpu time required in the analysis was 100 s for this case.

The soil-structure system was also analyzed by using the mode shapes of the total system for transforming the equations of motion. In all 10 modes were required to obtain an accuracy comparable to that obtained by using normal modes in association with constraint modes. This cpu time required in the analysis was 187 s.

It is evident from the results presented that a transformation which uses fixed-base normal modes in association with constraint modes is quite effective and that the method of artificial damping is successful in eliminating aliasing errors in a discrete Fourier Transform analysis.

SUMMARY AND CONCLUSIONS

A procedure is suggested for the analysis of soil-structure system, in which the system of equations is reduced by a transformation that uses a few normal modes of the structure on a fixed base in association with static constraint modes obtained by applying a unit displacement, in turn, along each of the interface degrees of freedom. Analysis is performed in the frequency domain using the method of artificial damping. An example is presented to show that the suggested transformation is quite effective. Also, the method of artificial damping significantly improves the efficiency of frequency domain analysis. Similar conclusions have been arrived at by Xia (1993) who presents an example in which the soil region is taken to be an infinite half space modelled by BEM.

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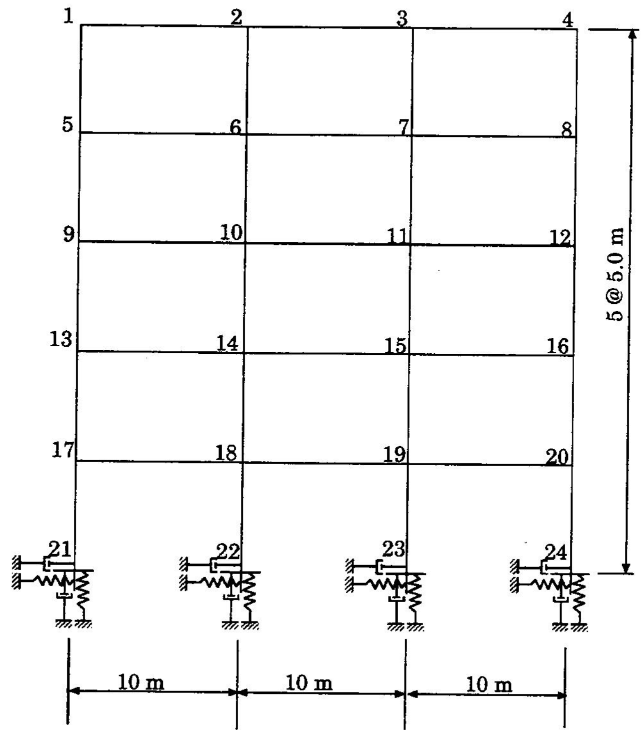
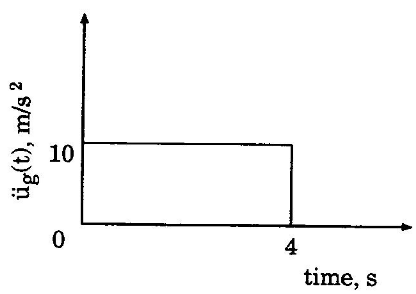


(a) Five storey frame



(b) Free field acceleration history

Fig. 1. Plane frame founded on flexible soil subjected to a rectangular free field acceleration

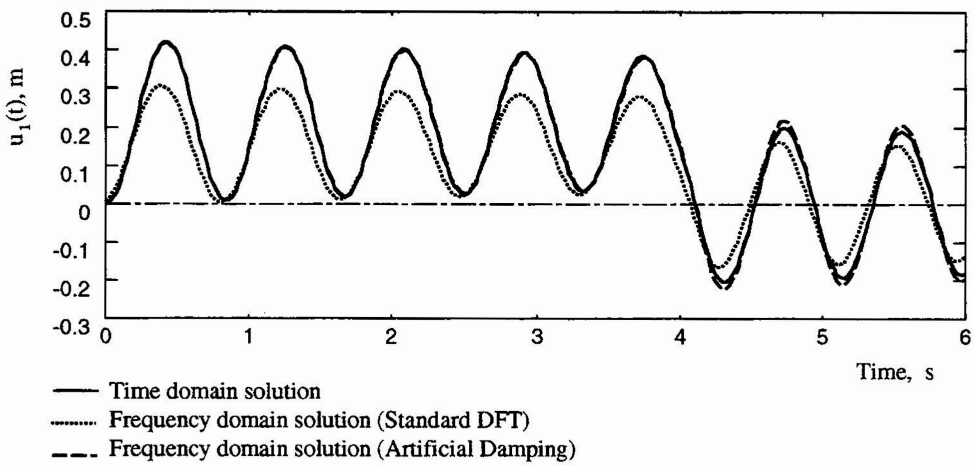


Fig. 2. Time and frequency domain solution of full structure.

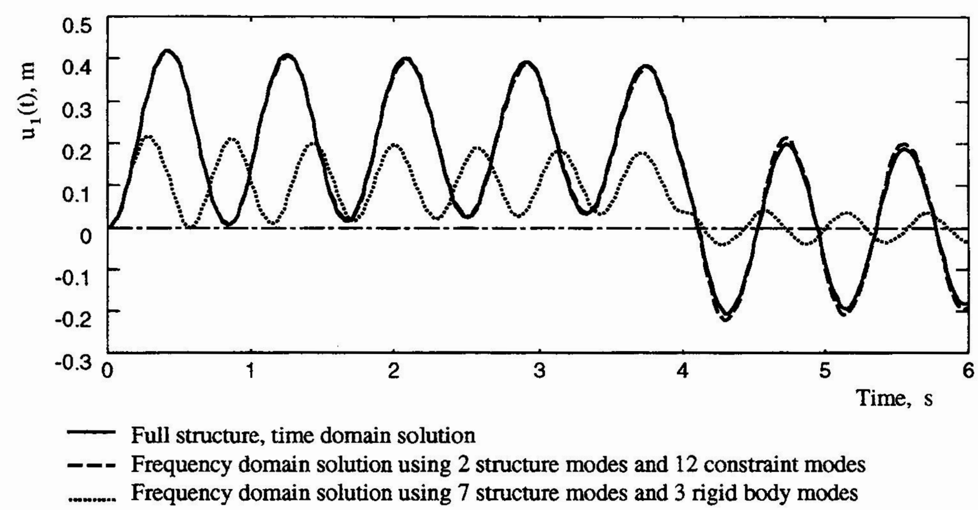


Fig. 3. Comaprison of the response of a reduced system with the exact response.